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FIXED POINT THEOREMS FOR CONTRACTIVE MAPPINGS OF INTEGRAL TYPE IN *n*-BANACH SPACES

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Abstract

In this paper, we present some common fixed point theorems for class of mappings satisfying contractive condition of integral type in *n*-Banach spaces. Our results are version of some known results.

1. Introduction

In [8, 9], Gähler introduced an attractive theory of 2-norm and *n*-norm on

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a linear space. Freese and Cho [7] gave as a survey of the latest results on the relations between linear 2-normed spaces and normed linear spaces and completion of linear 2-normed spaces. Later on Misiak [18] had also developed the notion of an n-norm in 1989. The concept on n-inner product spaces is also due to Misiak who had studied the same as early as 1980: spaces. A systematic development of linear n-normed spaces has been extensively made by Kim and Cho [15] and Malceski [17], Misiak [18] and Ganawan and Mashadi [13]. For related works of n-metric spaces and n-inner product spaces, see for example, [18], [11] and [13]. In 2002, Branciari [5] obtained a fixed point theorem for a single mapping satisfying an analogue of a Banach contraction principle for integral type inequality [4].

After the paper of Branciari, a lot of research works have been carried out on generalizing contractive conditions of integral type for different contractive mappings satisfying some properties, see for example, [1, 3, 4, 12, 14, 19, 16]. The aim of this paper is to transform the concept of fixed point in n-Banach spaces into fixed point in n-Banach spaces of integral type by using known contractive type mapping.

We recall some preliminary definitions.

Definition 1.1 [10]**.** Let n be a natural number, let X be a real vector space of dimension $d \geq n$ (d may be infinity). A real valued function $\| \cdot, \ldots, \cdot \|$ on X^n satisfying four properties:

(i) $\| x_1, \ldots, x_n \| = 0$ if and only if $x_1, \ldots, x_n$ are linearly dependent in X,

(ii) $\| x_1, \ldots, x_n \|$ is invariant under permutation of $x_1, x_2, \ldots, x_n$,

(iii) $\| x_1, \ldots, x_{n-1}, \alpha x_n \| = | \alpha | \| x_1, \ldots, x_{n-1}, x_n \|$ for every $\alpha \in R$,

(iv) $\| x_1, \ldots, x_{n-1}, y + z \| \leq \| x_1, \ldots, x_{n-1}, y \| + \| x_1, \ldots, x_{n-1}, z \|$ for all y and z in X, is called an *n-norm* over X and the pair $(X, \| \cdot, \ldots, \cdot \|)$ is called a *linear n-normed space*.

Example 1.1 [13]. Let $X = R^n$ with the norm $\| \cdot, ..., \cdot \|$ on X by

$$\| x_1, ..., x_n \| = | x_{ij} | = \left(\begin{vmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{vmatrix} \right),$$

where $x_i = x_{i1}, x_{i2}, ..., x_{in} \in R^n$ for each $i = 1, 2, ..., n.$ Then $(X, \| \cdot, ..., \cdot \|)$ is a linear n-normed space.

Definition 1.2 [10]. A sequence x_k in an n-normed space $(X, \| \cdot, ..., \cdot \|)$ is said to *converge* to an element $x \in X$ (in the n-norm) whenever $\lim_{k \to \infty} \| u_1, ..., u_{n-1}, x_k - x \| = 0$ for every $u_1, ..., u_{n-1} \in X.$

Definition 1.3 [19]. A sequence x_k in an n-normed space $(X, \| \cdot, ..., \cdot \|)$ is said to be *Cauchy sequence* with respect to n-norm if $\lim_{k \to \infty} \| u_1, ..., u_{n-1}, x_k - x \| = 0$ for every $u_1, ..., u_{n-1} \in X.$

Definition 1.4 [10]. If every Cauchy sequence in X converges to an element $x \in X,$ then X is said to be *complete* (with respect to the n-norm). A complete n-normed space is called an *n-Banach space*.

Definition 1.5 [10]. Let X be an n-Banach space and T be a self mapping of X. T is said to be *continuous* at x if for every sequence x_k in X, $x_k \to x$ as $k \to \infty$ implies $Tx_k \to Tx$ as $k \to \infty$ in X.

2. Main Results

Before starting the main results, first we write the following definition:

Definition 2.1 [2]. $\varphi : [0, +\infty] \to [0, +\infty]$ is subadditive on each $[a, b] \subset [0, +\infty)$ if,

$$\int_0^{a+b} \varphi(t)dt \le \int_0^a \varphi(t)dt + \int_0^b \varphi(t)dt.$$

Theorem 2.1. *Let f be a self mapping of n-Banach space X such that*

$$\int_0^{\| fx-fy, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$\leq \alpha\int_0^{\| x-y, u_1, ..., u_{n-1} \|} \varphi(t)dt + \beta\int_0^{\| x-fx, u_1, ..., u_{n-1} \| + \| y-fy, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$+ \gamma\int_0^{\| x-fy, u_1, ..., u_{n-1} \| + \| y-fx, u_1, ..., u_{n-1} \|} \varphi(t)dt \tag{2.1}$$

for each $x, y, u_1, ..., u_{n-1} \in X$ *with non negative reals* $\alpha + 2\beta + 2\gamma < 1$, *where* $\varphi : [0, +\infty] \to [0, +\infty]$ *is a Lebesgue integrable mapping which is summable, subadditive on each* $[a, b] \subset [0, +\infty]$, *non-negative and for each*

$$\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dt > 0. \tag{2.2}$$

Then f has a unique fixed point in X, with $\lim_{k\to\infty} f^k x_\circ = z$ *for each* $x_\circ \in X$.

Proof. Let $x_\circ \in X$, and define the iterate sequence x_k by

$$x_{k+1} = fx_k = f^{k+1}x. \tag{2.3}$$

Then by (2.1) and (2.3), we get

$$\int_0^{\| x_k - x_{k-1}, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$= \int_0^{\| fx_{k-1} - fx_k, u_1, ..., u_{n-1} \|} \varphi(t)dt \leq \alpha\int_0^{\| x_{k-1} - x_k, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$+ \beta\int_0^{\| x_{k-1} - fx_{k-1}, u_1, ..., u_{n-1} \| + \| x_k - fx_k, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$+ \gamma\int_0^{\| x_{k-1} - fx_k, u_1, ..., u_{n-1} \| + \| x_k - fx_{k-1}, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$\leq \left(\frac{\alpha + \beta + \gamma}{1 - \beta - \gamma}\right)\int_0^{\| x_{k-1} - x_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$= q\int_0^{\| x_{k-1} - x_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt,$$

where

$$q = \left(\frac{\alpha + \beta + \gamma}{1 - \beta - \gamma}\right),$$

for each k and for $u_1, \ldots, u_{n-1} \in X$.

Proceeding in this way, we have

$$\int_0^{\| x_k - x_{k-1}, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt \leq q^2 \int_0^{\| x_{k-2} - x_{k-1}, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq \cdots \leq q^n \int_0^{\| x_\circ - x_1, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt,$$

where

$$q = \frac{\alpha + \beta + \gamma}{1 - \beta - \gamma} < 1.$$

If $m > k$, for $u_1, \ldots, u_{n-1} \in X$, we get

$$\int_0^{\| x_k - x_m, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq \int_0^{\| x_k - x_{k+1}, u_1, \ldots, u_{n-1} \|} \varphi(t) + \int_0^{\| x_{k+1} - x_{k+2}, u_1, \ldots, u_{n-1} \|} \varphi(t)$$

$$+ \cdots + \int_0^{\| x_{m-1} - x_m, u_1, \ldots, u_{n-1} \|} \varphi(t)$$

$$\leq (q^k + q^{k+1} + \cdots + q^{m-1})\int_0^{\| x_\circ - x_1, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq q^k(1+q+q^2+\cdots+q^{m-k-1})\int_0^{\| x_\circ - x_1, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt$$

$$\leq \frac{q^k}{1-q}\int_0^{\| x_\circ - x_1, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt \to \infty \quad \text{as} \quad k \to \infty.$$

That means x_k is a Cauchy sequence in *X*, and let $\lim_{k\to\infty} x_k = z \in X$.

Again for $u_1, \ldots, u_{n-1} \in X$, we have

$$\int_0^{\| x_{k+1} - fz, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt$$

$$= \int_0^{\| fx_k - fz, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt \leq \alpha\int_0^{\| x_k - z, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt$$

$$+ \beta\int_0^{\| x_k - x_{k-1}, u_1, \ldots, u_{n-1} \| + \| z - fz, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt$$

$$+ \gamma\int_0^{\| x_k - fz, u_1, \ldots, u_{n-1} \| + \| z - x_{k+1}, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt,$$

taking limit as $k \to \infty$, we have

$$\int_0^{\| z - fz, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt \leq (\beta+\gamma)\int_0^{\| z - fz, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt,$$

since $\beta + \gamma < 1$, we get a contradiction, therefore $\int_0^{\| z - fz, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt = 0$, by using (2.2), we obtain that $z = fz$ and *z* is a fixed point of *f*, where $\lim_{k\to\infty} f^k x_\circ = z$ for each $x_\circ \in X$. To prove the uniqueness of *z*, suppose that $(w \neq z)$ be another fixed point of *f*, then from (2.1), we get

$$\int_0^{\| z - w, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt$$

$$= \int_0^{\| fz - fw, u_1, \ldots, u_{n-1} \|}\varphi(t) \leq \alpha\int_0^{\| z - w, u_1, \ldots, u_{n-1} \|}\varphi(t)\,dt$$

$$\leq \beta \int_0^{\| z-fz, u_1, \ldots, u_{n-1} \| + \| w-fw, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \gamma \int_0^{\| z-fw, u_1, \ldots, u_{n-1} \| + \| w-fz, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq (\alpha + 2\gamma) \int_0^{\| z-w, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

since $\alpha + 2\gamma < 1$, we get a contradiction, therefore $\int_0^{\| z-w, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$ $= 0$, by using (2.2), we obtain that $z = w$. Therefore, z is a unique fixed point of *f*.

Remark 2.1. (i) On setting $\varphi(t) = 1$ over R^+, the contractive condition of integral type transforms into a general contractive condition not involving integral.

(ii) From Condition 2.1 of integral type several contractive mappings of integral type can be obtained.

Now, our next theorem is the extension of Theorem 2.1 for a pair of mappings.

Theorem 2.2. *Let f and g be self mappings of n-Banach space X satisfying for all* $u_1, \ldots, u_{n-1} \in X$,

$$\int_0^{\| fx-gy, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq \alpha \int_0^{\| x-y, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \beta \int_0^{\| x-fx, u_1, \ldots, u_{n-1} \| + \| y-gy, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \gamma \int_0^{\| x-gy, u_1, \ldots, u_{n-1} \| + \| y-fx, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt \tag{2.4}$$

for each $x, y, u_1, ..., u_{n-1} \in X$ *with non negative reals* $\alpha + 2\beta + 2\gamma < 1$, *where* $\varphi : [0, +\infty] \to [0, +\infty]$ *is a Lebesgue integrable mapping which is summable, subadditive on each* $[a, b] \subset [0, +\infty]$, *non-negative and for each* $\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dt > 0.$

Then f and g have a unique fixed point in X, with $\lim_{k\to\infty} f^k x_\circ = z$ *for each* $x_\circ \in X$.

Proof. Let $x_\circ \in X$, and define the iterate sequence x_k by

$$x_{2k+1} = fx_k \text{ and } x_{2k+2} = gx_{k+1}.$$

Then by (2.4), we have

$$\int_0^{\| x_{2k+1}-x_{2k+2}, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$= \int_0^{\| fx_{2k}-gx_{k+1}, u_1, ..., u_{n-1} \|} \varphi(t)dt \le \alpha \int_0^{\| x_{2k}-x_{2k+1}, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$+ \beta \int_0^{\| x_{2k}-x_{2k+1}, u_1, ..., u_{n-1} \| + \| x_{2k+1}-x_{2k+2}, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$+ \gamma \int_0^{\| x_{2k}-x_{2k+2}, u_1, ..., u_{n-1} \| + \| x_{2k+1}-x_{2k+1}, u_1, ..., u_{n-1} \|} \varphi(t)dt,$$

by the simple calculation, we get

$$\int_0^{\| x_{2k+1}-x_{2k+2}, u_1, ..., u_{n-1} \|} \varphi(t)dt$$

$$\le \left(\frac{\alpha + \beta + \gamma}{1 - \beta - \gamma}\right) \int_0^{\| x_{2k}-x_{2k+1}, u_1, ..., u_{n-1} \|} \varphi(t)dt,$$

by exactly the same argument we produce

$$\int_0^{\| x_{2k}-x_{2k+1}, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt \leq q\int_0^{\| x_{2k-1}-x_{2k}, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt,$$

for all $x,\ y,\ u_1,\ \ldots,\ u_{n-1} \in X$ and for all k, we get

$$\int_0^{\| x_k-x_{k+1}, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt \leq q\int_0^{\| x_{k-1}-x_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq q^2\int_0^{\| x_{k-2}-x_{k-1}, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\cdots \leq q^k\int_0^{\| x_\circ-x_1, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt,$$

by the same steps of Theorem 2.1, one can reach to conclude that x_k is a Cauchy in X, and let $\lim\limits_{k\to\infty} x_k = z \in X$.

Now, for $u_1,\ \ldots,\ u_{n-1} \in X$, by (2.4), we have

$$\int_0^{\| x_{2k+1}-gz, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$= \int_0^{\| fx_{2k}-gz, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt \leq \alpha\int_0^{\| x_{2k}-z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \beta\int_0^{\| x_{2k}-x_{2k+1}, u_1, \ldots, u_{n-1} \| + \| z-gz, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \gamma\int_0^{\| x_{2k}-gz, u_1, \ldots, u_{n-1} \| + \| z-x_{2k+1}, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt.$$

Taking the limit as $k \to \infty$, gives

$$\int_0^{\| z-gz, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt \leq (\beta + \gamma)\int_0^{\| z-gz, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt,$$

since $(\beta + \gamma < 1)$ we get $\int_0^{\| z-gz, u_1, \ldots, u_{n-1} \|} \varphi(t) dt = 0,$ using (2.2), we conclude that $z = gz.$ Similarly, we can show that $z = fz,$ and hence z is a common fixed point of f and g.

Next, suppose that $(w \neq z)$ is another common fixed point of f and g, then from (2.4), we get

$$\int_0^{\| z-w, u_1, \ldots, u_{n-1} \|} \varphi(t) dt$$

$$= \int_0^{\| fz-gw, u_1, \ldots, u_{n-1} \|} \varphi(t) \leq \alpha \int_0^{\| z-w, u_1, \ldots, u_{n-1} \|} \varphi(t) dt$$

$$\leq \beta \int_0^{\| z-fz, u_1, \ldots, u_{n-1} \| + \| w-gw, u_1, \ldots, u_{n-1} \|} \varphi(t) dt$$

$$+ \gamma \int_0^{\| z-gw, u_1, \ldots, u_{n-1} \| + \| w-fz, u_1, \ldots, u_{n-1} \|} \varphi(t) dt$$

$$\leq (\alpha + 2\gamma) \int_0^{\| z-w, u_1, \ldots, u_{n-1} \|} \varphi(t) dt,$$

since $\alpha + 2\gamma < 1,$ we get a contradiction, therefore $\int_0^{\| z-w, u_1, \ldots, u_{n-1} \|} \varphi(t) dt$ $= 0,$ by using (2.2), we obtain that $z = w.$ Therefore, z is a unique common fixed point of f and g. The proof is complete.

Finally, we extend the result for a sequence of mappings.

Theorem 2.3. *Let X be n-Banach space with $f : X \to X$ and $f_k : X \to X$, be a sequence of mappings such that*

(i)

$$\int_0^{\| f_k x - f_k y, u_1, \ldots, u_{n-1} \|} \varphi(t) dt$$

$$\leq \alpha\int_0^{\| x-y, u_1, ..., u_{n-1} \|} \varphi(t)\,dt$$

$$+\,\beta\int_0^{\| x-f_k x, u_1, ..., u_{n-1} \| + \| y-f_k y, u_1, ..., u_{n-1} \|} \varphi(t)\,dt$$

$$+\,\gamma\int_0^{\| x-f_k y, u_1, ..., u_{n-1} \| + \| y-f_k x, u_1, ..., u_{n-1} \|} \varphi(t)\,dt, \qquad (2.5)$$

(ii) $\lim_{k\to\infty} f_k x = fx$ *for each* $x \in X$,

for each $x, y, u_1, ..., u_{n-1} \in X$ *with non negative reals* $\alpha + 2\beta + 2\gamma < 1$, *where* $\varphi : [0, +\infty] \to [0, +\infty]$ *is a Lebesgue integrable mapping which is summable, subadditive on each* $[a, b] \subset [0, +\infty]$, *non-negative and for each* $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)\,dt > 0$. *Then f has a unique fixed point in X, such that* $\lim_{k\to\infty} z_k = z$, z_k *being the unique fixed point of* f_k, $k = 1, 2, \ldots$.

Proof. If we take the limit in (2.5), we have

$$\int_0^{\| fx-fy, u_1, ..., u_{n-1} \|} \varphi(t)\,dt$$

$$\leq \alpha\int_0^{\| x-y, u_1, ..., u_{n-1} \|} \varphi(t)\,dt$$

$$+\,\beta\int_0^{\| x-fx, u_1, ..., u_{n-1} \| + \| y-fy, u_1, ..., u_{n-1} \|} \varphi(t)\,dt$$

$$+\,\gamma\int_0^{\| x-fy, u_1, ..., u_{n-1} \| + \| y-fx, u_1, ..., u_{n-1} \|} \varphi(t)\,dt$$

for all $x, y, u_1, ..., u_{n-1} \in X$ and hence *f* satisfies (2.1). Hence by Theorem 2.1, *f* has a unique fixed point say $z \in X$. Now for all $u_1, ..., u_{n-1} \in X$

$$\int_0^{\| z-z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$= \int_0^{\| fz-f_k z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq \int_0^{\| fz-f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt + \int_0^{\| f_k z-f_k z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt, \qquad (2.6)$$

again from (2.5), we obtain

$$\int_0^{\| f_k z-f_k z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq \alpha \int_0^{\| z-z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \beta \int_0^{\| z-f_k z, u_1, \ldots, u_{n-1} \| + \| z_k-f_k z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \gamma \int_0^{\| z-f_k z_k, u_1, \ldots, u_{n-1} \| + \| z_k-f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt,$$

then, by the condition (ii) of this theorem and taking the limit as $k \to \infty$, we get

$$\int_0^{\| f_k z-f_k z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\leq \alpha \int_0^{\| z-z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt + \beta \int_0^{\| z-f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \gamma \int_0^{\| z-z_k, u_1, \ldots, u_{n-1} \| + \| z_k-f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt, \qquad (2.7)$$

from (2.7) in (2.6), we have

$$\int_0^{\| z-z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$= \int_0^{\| fz - f_k z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt \le \int_0^{\| fz - f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \alpha \int_0^{\| z - z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt + \beta \int_0^{\| z - f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \gamma \int_0^{\| z - z_k, u_1, \ldots, u_{n-1} \| + \| z_k - f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$\le \left(\frac{1 + \beta}{1 - \alpha - \gamma} \right) \int_0^{\| z - f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt$$

$$+ \left(\frac{\gamma}{1 - \alpha - \gamma} \right) \int_0^{\| z_k - f_k z, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt.$$

So, by condition (ii) of this theorem, we get a contradiction, therefore

$$\int_0^{\| z - z_k, u_1, \ldots, u_{n-1} \|} \varphi(t)\,dt \le 0, \text{ as } k \to \infty.$$

So, by (2.2), we get $\lim_{k \to \infty} z_k = z$ (this completes the proof).

Remark 2.2. The results in this paper also hold for contractive mappings of integral type in 2-Banach spaces.

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